Carsten Berkau

Management Accounting
7^{th} Edition

- Link 14.A –

Common Format for Simple Regression Method

The common format for the simple regression method is based on the variables X and Y. The simple regression method determines a linear function Y(X) which has the form Y(X) = a × X + b. a is the slope of the line and b is the value for X = 0. Y(0) = b.

For n observations, indicated by i = 1 … n, the two equations look as below:

$$(1)\quad \sum_{i=1}^{n} X_i \times Y_i = b \times \sum_{i=1}^{n} X_i + a \times \sum_{i=1}^{n} X_i^2$$

$$(2)\quad \sum_{i=1}^{n} Y_i = n \times b + a \times \sum_{i=1}^{n} X_i$$

Regression methods help us to find and analyse dependencies between characteristics, such as prices, and independent numbers. Assume you get hold of a data set of prices obtained for property sales during the last year in your neighbourhood. Your property manager might also provide you with characteristics of the sold houses, such as area in square metres, plot size, age, distance to the next school etc. In this case the regression method can be applied to determine a price function that depends on various parameters (square metres, plot size, age, distance to the next school etc.). Once you know the dependencies and your house's parameters, you can calculate its selling price based on the observations from in the past. Computer programs that provide you with used-car valuations like the German Schwacke-Liste, are based on regression methods and data input from recent sales.
In contrast, in Accounting we only discuss a cost function depending on a single variable: the output, such as the tax statements at DANNING (Pty) Ltd. Because the simple regression method determines one dependency only, we classify it to 'simple'.
Secondly, we want to explain the graphical interpretation of the simple regression method:
The aim is to determine 2 parameters of a linear function Y(X), which are the slope and the intersection with the Y-axis. It would be the parameters a and b, or with regard to the DANNING (Pty) Ltd. case study, the proportional costs PC and the fixed costs FC. For the calculation of two parameters, 2 independent equations are necessary. The first one represents areas whereas the second equation is linked to the Y-values, in DANNING (Pty) Ltd.'s case to costs.
The equations are explained below based on only 2 observations. Accordingly, the parameter for the total has been adjusted. n is now replaced by 2.

$$(1)\ \sum_{i=1}^{2} X_i \times Y_i = b \times \sum_{i=1}^{2} X_i + a \times \sum_{i=1}^{2} X_i^2$$

On the left side of the equation there are two areas in the shape of rectangles. The total of the areas equals: $X_1 \times Y_1 + X_2 \times Y_2$.

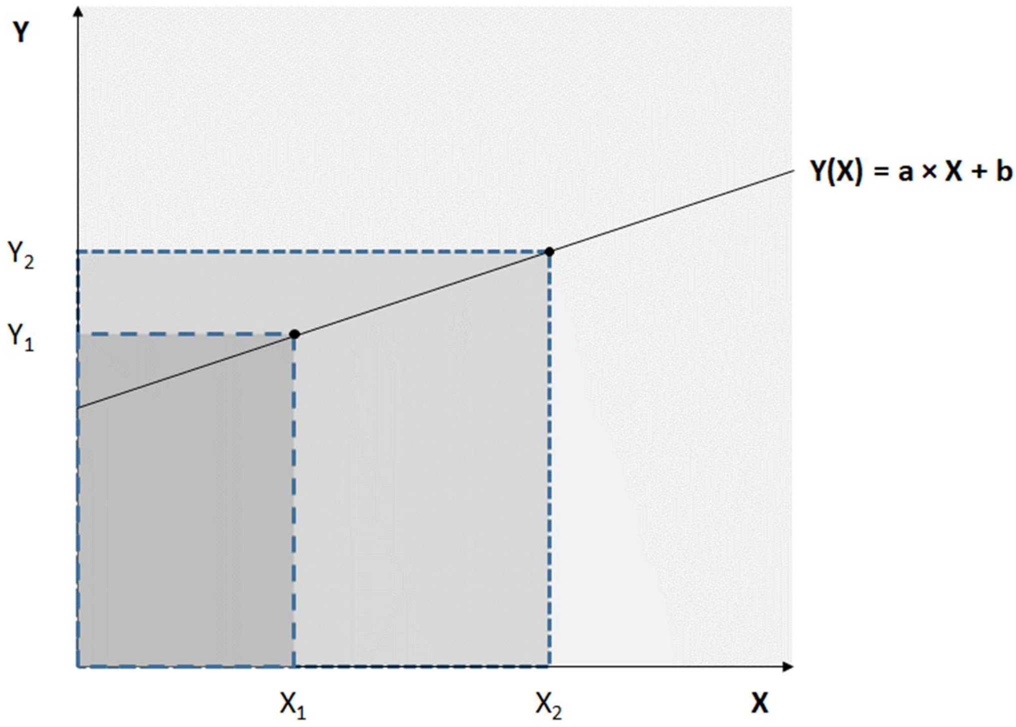

Figure 1: Marked areas in the linear function Y(X)

In Figure 1 the area $X_1 \times Y_1$ is marked as a dotted line with a dark shading and the area $X_2 \times Y_2$ by a light shading. The areas are overlapping.
In the diagram the function is drawn as a line and indicated by: $Y(X) = a \times X + b$.
a is the slope of the line. We describe the slope of the function Y(X) by a with: $a = (Y_2 - Y_1) / (X_2 - X_1)$.
As we can see in Figure 2 the slope is the tangent of the angle α which is the opposite leg of the angle α divided by its adjacent leg.

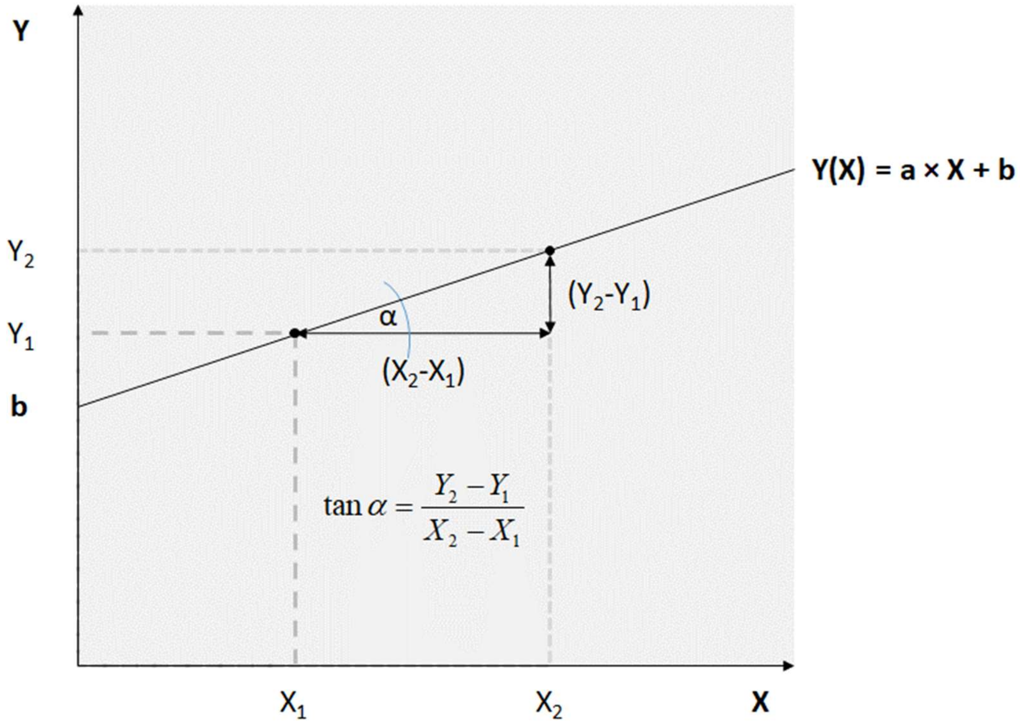

Figure 2: calculation of the slope of Y(X)

By the next step we study the right-hand side of equation (1). It contains 4 areas: A, B, C and D. We study the areas A and C at first which result from the first observation.

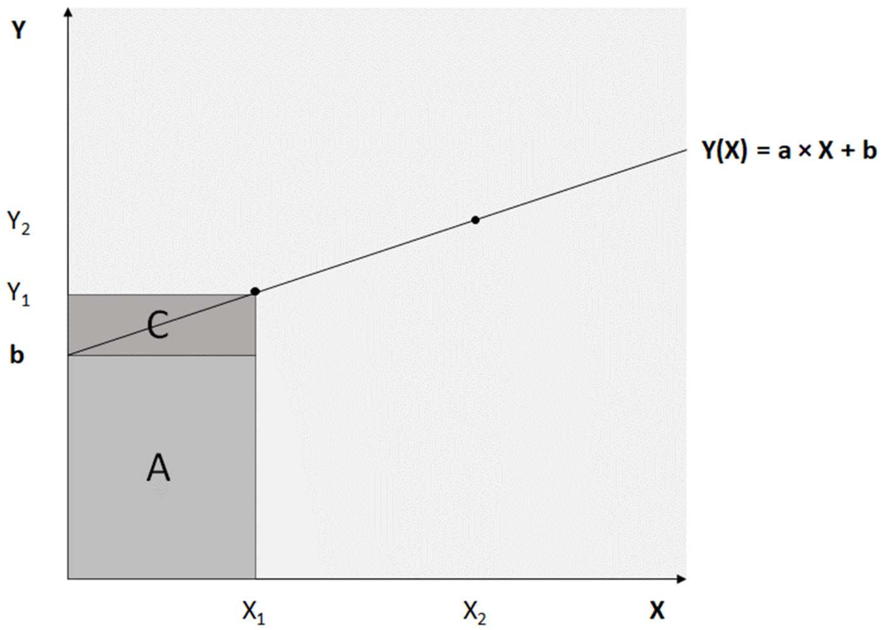

Figure 3: Areas A and C for the function Y(X)

The area A got the size: $b \times X_1$. Regarding the formula, the area C has the size: $a \times X_1^2$ multiplied by a factor, see below. The area A forms a square

if a = tan ∝ equals 1. This will be the case if: ∝ = 45° because tan 45° = 1. This means graphically that the length of (Y_1 – b) is equal to the one of (X_1 – 0).
In cases the angle ∝ is lower than 45° the tangent "squeezes" the area. E.g., if ∝ = 30° a becomes: a = tan 30° = **0.57**. As a result, the square area is multiplied with the factor 57%. This will squeeze the square and the area C becomes a flat rectangle. This applies for the area C in Figure 4.

In all cases where the angle ∝ exceeds 45°, e.g., if ∝ = 60°, the slope is steep and the factor a will stretch area C in an upwards manner. At tan 60° = **1.73** the area C will appears as an upright rectangle.
The area C depends on the slope of the function. We expressed it by a. For the 2nd observation the same rule applies. Compare to Figure 4. The factor a applies for the area C and D calculation the same way.

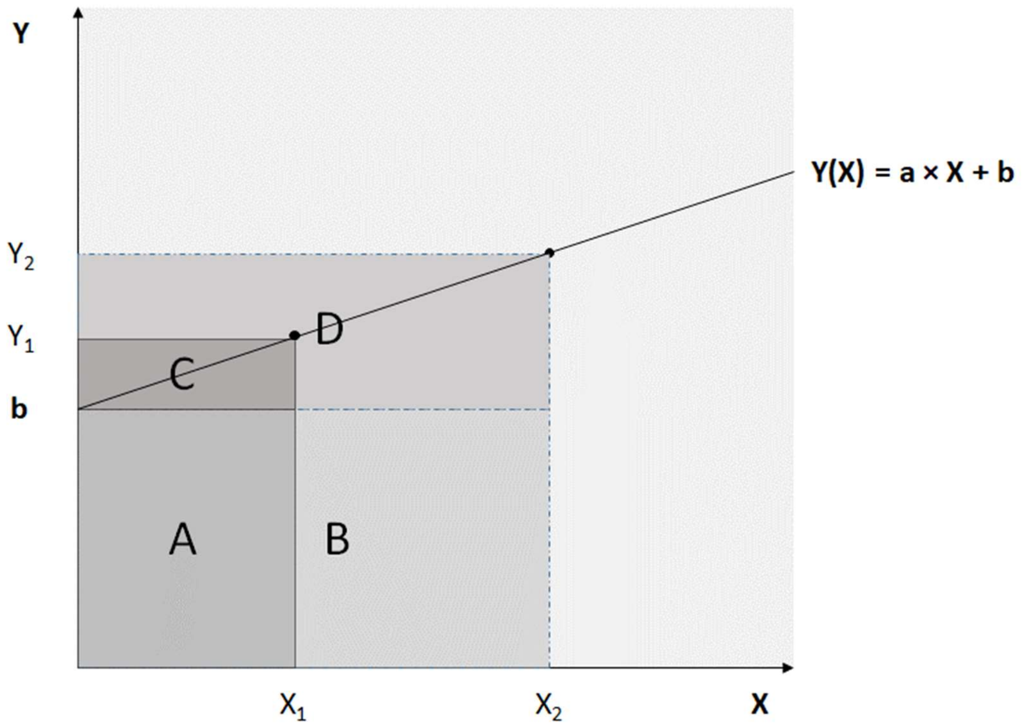

Figure 4: Areas A and C as well as B and D for the function Y(X)

If you compare the total of the areas A, B, C and D you will see that these areas equal those depicted in Figure 1. As a result, the equation below is valid:

$X_1 \times Y_1 + X_2 \times Y_2 = A + B + C + D$
$= b \times (X_1 + X_2) + \tan \alpha \times (X_1^2 + X_2^2)$

We now study the equation (2) for 2 observations:

$$(2)\ \sum_{i=1}^{2} Y_i = 2 \times b + a \times \sum_{i=1}^{2} X_i$$

On the left-hand side of the equation, the total of Y_1 and Y_2 is calculated. In case of DANNING (Pty) Ltd., the Y-values represent costs for the monthly observations Y_1 and Y_2.
On the right-hand side, there are two terms. The first one represents the portion of Y-values that does not depend on Y. Therefore, the first terms got the height of: $2 \times b$. The second term is the value of $(Y_1 – b)$ and $(Y_2 – b)$ that depends proportionally on X_i.

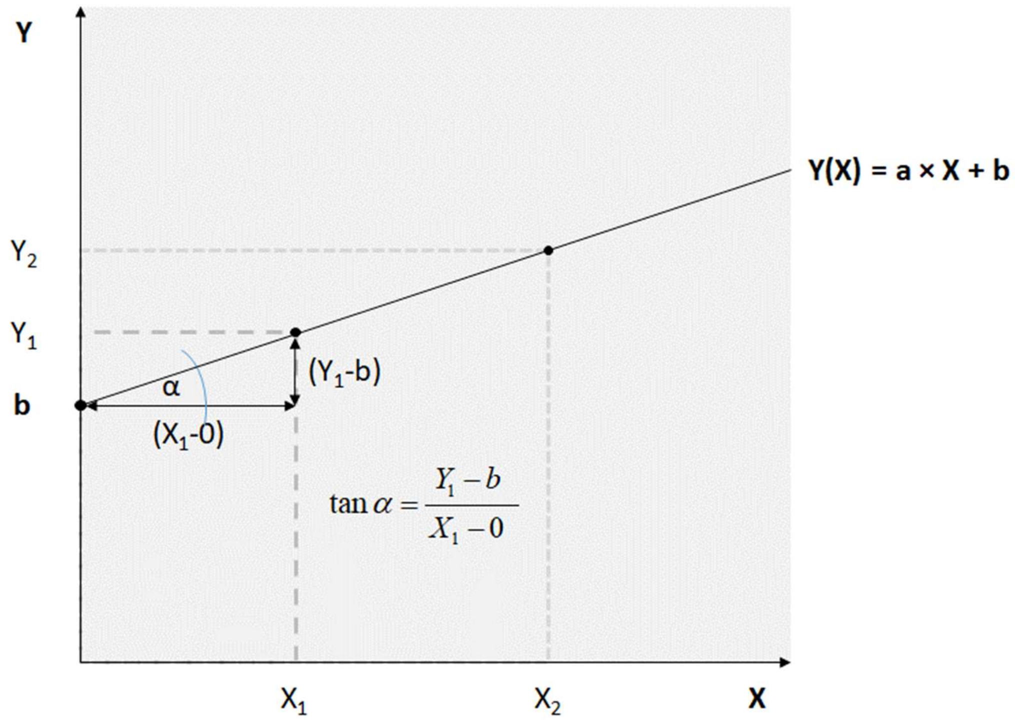

Figure 5: Function Y(X) = a × X +b

The slope of the function Y(X) is a and can be calculated as $a = \tan \alpha = (Y_1 – b) / X_1$. Check Figure 5. The same applies for the second mark on the function that represents the second observation: $a = \tan \alpha = (Y_2 – b) / X_2$.
If the slope of the function is positive the contribution for the second observation to the Y-value will be higher than for the first one because $Y_i – b = a \times X_i = \tan \alpha \times X_i$.
The equation below is valid:

$$Y_1 + Y_2 = 2 \times b + \tan a \times (X_1 + X_2)$$

The simple regression method is an instrument widely applied in Management Accounting to separate costs for cost functions that depend linear from a reference unit.